Introduction of Artificial Boundary Conditions in the Spectral Moments Method

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***Abstract*—The overall objective of this paper is to demonstrate the ability of the spectral moments method, a new method in electromagnetism, to incorporate absorbing boundary conditions. This demonstration is done successfully with plane pulse propagation in free space, and through a comparison with finite-difference time-domain (FDTD) results. The very good agreement of the results leads to the conclusion that the spectral moments method application for electromagnetic propagation and diffraction problems should be further investigated.**

I. Introduction

THIS letter deals with the application of a method called "Spectral Moments Method" (SMM) to the simulation of electromagnetic wave propagation and diffraction. This method is new in electromagnetism; it is a mixed method which works both in time and in frequency domain and directly gives the result for all frequencies and any incident pulse, and which can cope with very large computational volumes. Previous works have shown the efficiency of this method when applied to electromagnetic problems, but still the implementation of absorbing boundary conditions (ABC's) had not been done in detail. This is the object of this paper. After introducing the method in the first part, we present ABC. Then, the results of the implementation in free space and a comparison with finite-difference time-domain (FDTD) are shown in the time domain.

II. The Spectral Moments Method

This numerical method, developed first in condensed matter physics [1], [2], was recently successfully applied to the propagation simulation of acoustic waves in geophysics [3]. The mathematical basis of this method has been developed by Benoit [4]. First applications concerning electromagnetic waves propagation in anisotropic media was presented by Lakhliai [5]. Diffraction by a circular and square cylinder has recently been reported by Poussigue [6] and Chenouni [7].

In the SMM, the space variables only are sampled, and getting the solution at any frequency requires only one computation. It is based on determination of the exact Green function of the system and is mathematically equivalent to the Lanczos procedure, but much more general. The method is widely developed in [4] and [5], so we will only succinctly describe the principle. The problem we want to solve at hand is: assuming a dielectric media and an external source, what is the value of the electric field $\vec{E}$ and of the magnetic field $\vec{B}$ in a point $\vec{r}$ at time t? After spatial discretization and a simple change of variables, Maxwell's equations can be expressed in the following form:

$$\frac{\partial \vec{X}}{\partial t} + iM\vec{X} = -\vec{J}'$$

where $i^2 = -1$. The discretization grating is composed of N cells, $\vec{X}$ is a $6 \times N$ components vector representing the components of the electric and magnetic fields $\vec{E}$ and $\vec{B}$ at each node of the grid, and M is a sparse matrix which corresponds to the geometry of the problem. The size of M is $6 \times N$, but in practice it is never stored totally. $\vec{J}'$ is a $6 \times N$ source vector. Let us introduce the Green matrix G solution of

$$\frac{dG(t-t')}{dt} + iMG(t-t') = -I\delta(t-t')$$

where I is the identity matrix. After Fourier transform, we obtain in the frequency domain

$$G(\omega) = -i(\omega I - M)^{-1}.$$

Then we can directly deduce the solution

$$\vec{X}(\omega) = G(\omega) \cdot \vec{J}'(\omega)$$

The whole problem is then the computation of the Green function $G(\omega)$, which takes into account the boundary conditions of the system. The Fourier transform of a Green function between a α polarized response at the node n, and a β polarized source in the node n' is given by the matrix element

$$G_{\alpha\beta}(n, n', \omega) = -i(I\omega - M)^{-1}_{\alpha n \beta n'}.$$

We can prove that $G_{\alpha\beta}(n, n', \omega)$ can be obtained from a continued fraction expansion versus ω, and the coefficients of this expansion are directly calculated from the dynamic matrix M [2]. The problem is the convergence of this continued fraction: if there is no conductivity, M is real and symmetric, and a mathematical result [8] ensures this convergence; but if there is some conductivity, then M is complex and no longer symmetric. In that case, there is no theoretical result, neither on the stability of the fraction coefficients nor on the convergence of the fraction. Nevertheless, in order to study diffraction problems with the SMM, it was really necessary to know if ABC could be introduced in the method.

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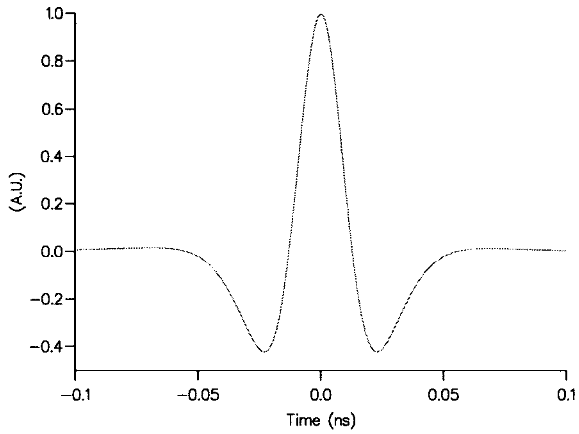

Fig. 1. Incident pulse.

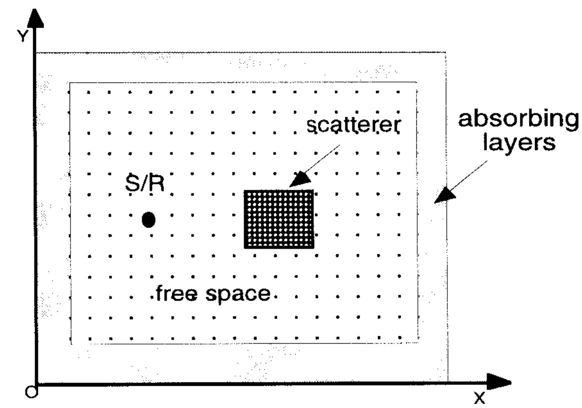

Fig. 2. Computational domain.

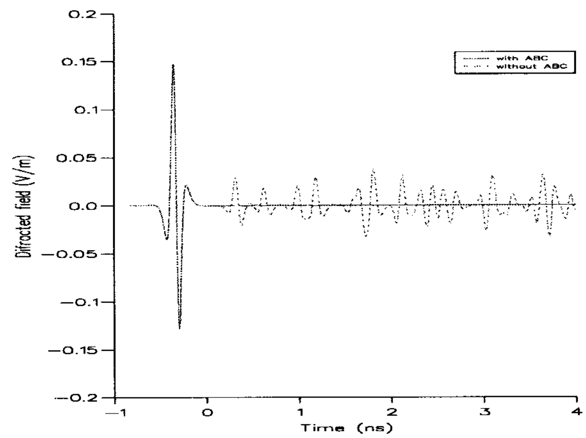

Fig. 3. Propagation of a cylindrical wave, with or without ABC's.

III. Absorbing Boundary Conditions

A drawback of spatial discretization methods lies in the fact that Maxwell's equations have to be solved in a discretized domain whose size needs to be limited for memory purposes. Nevertheless, open problems involving theoretically boundless space extension can be solved when applying special conditions on the boundaries of the computational domain, in order to absorb outgoing waves. Many ABC's are available in literature: perfectly matched layer method [9], Bayliss and Turkel asymptotic operators method [10], etc.

For easy implementation, we use broadband Radar Absorbing Material, which consists of an addition of absorbing layers all around the truncated domain, and we terminate with a metallic wall. Maxwell's equations then become

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \rho \vec{H} \quad \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \sigma \vec{E}$$

where ρ and σ are magnetic and electric conductivity, which depends on position and are such as $\rho = \sigma \cdot \eta^2$ where η is the intrinsic impedance of the medium contiguous to the ABC. We can easily show that these equations lead to the problem $(\partial \vec{X}/\partial t) + iM\vec{X} = -\vec{J}'$ treated above. The main difference is that as we said in the previous section, M is no longer symmetric. Theory is then more complex [4], but it provides the same result: the solution is given by $\vec{X}(\omega) = G(\omega) \cdot \vec{J}'(\omega)$ where G elements can be obtained from a continued fraction expansion.

Now we study the numerical influence of such a change in the fraction coefficients stability, and the fraction convergence in this case.

IV. Results

The source (Fig. 1) we use is a Rayleigh-like plane pulse defined as follow in the xOy plane [11]:

$$S(x,t) = \mathrm{Re}\left[i\left(\frac{f}{2\pi}\left(t - \frac{x}{c}\right) + i\right)^{-5}\right] \text{where } f = 5 \text{ GHz}.$$

Though all combinations of sources and detectors have been treated (plane or point), results we present here concern near-field detection, with a point source and a point receiver. We close the computational domain with successive layers, one cell wide, incorporating electric and magnetic conductivities (cf., Fig. 2). After many optimization steps, best results were obtained with nine layers and the following relation:

$$\begin{aligned}\sigma_n &= 0.01 \times n^2 \quad \text{where } n = 1, \cdots, 9 \\ \rho_n &= \sigma_n \cdot \eta_0^2\end{aligned}.$$

In the SMM, use of very large conductivities increases the oscillation behavior of the coefficients of the continued fraction, and might introduce numerical instability of the solution [4]. The above values present the best tradeoff between tapering and efficiency of the layers on one hand, and stability of the results on the other hand.

To observe ABC efficiency, the first curves are simulating propagation of a cylindrical wave in free space. The domain is a 0.2-m size square, the receiver and the source are located in (0.05, 0.1). We can see in Fig. 3 that there is a total absorption of the incident wave when the layers are present.

We wanted to check that, on a ringing target, the method could account for weak resonances without generating spurious reflections; so the second result is the cylindrical wave diffraction by a dielectric cylinder ($\varepsilon_r = 9$). The cylinder is at the center of the domain, and its radius is 0.015 m. In order to validate the SMM for this case, we have compared the result with the one obtained with the FDTD algorithm for the same geometry. Fig. 4 shows the perfect agreement of the

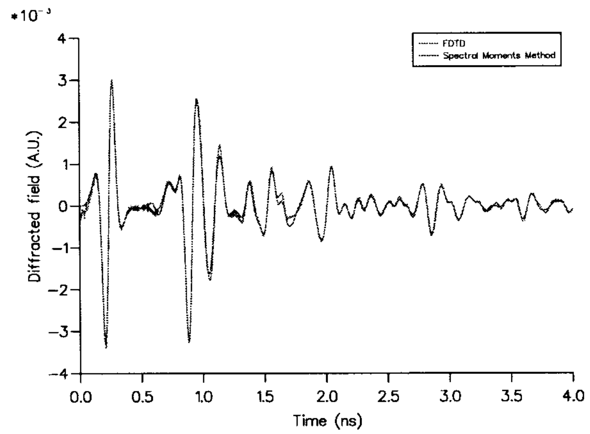

Fig. 4. Comparison between the spectral moments method and the FDTD method.

two curves; the only signal here is due to the resonances of the dielectric cylinder.

A convergence study of the continued fraction has shown that a reasonable number of coefficients [4] was sufficient to give a stable result. For the examples above, we needed 300 coefficients for the first case, and 1000 for the second one.

Notice that further optimization and systematic studies are necessary to make serious comparison between the SMM and the FDTD method efficiencies, particularly in the frequency domain.

V. Conclusion

We have shown that it is possible to introduce ABC in the SMM, and the results we obtain are very promising. We have used simple ABC and demonstrated the ability of the method to incorporate such boundary conditions. Consequently, others could be introduced as well (PML for example).

Through this introduction, we have shown the validity of the method on domains of reasonable size, and we can say now that it is worth taking interest in the spectral moments method application to propagation and diffraction problems.

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